



■ pqNTRUSign: update and recent results

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 BoardSecurity



- You may know us as **Security Innovation** ...
 - who owns IP of NTRU
- NTRUEncrypt patent released to public domain – March 2017
- Security Innovation (Embed business unit)



▣ Modular lattice signature scheme

- For a given L and a document m , its signature is a vector v
 - $v \in L$ can be generated efficiently with a trapdoor of L
 - $v = \text{hash}(m|L) \bmod p$ for some p
 - Forgery = solving approx.-SVP for L
- pqNTRUSign is a modular lattice signature with $L = \text{NTRU}$ lattice
 - Easy construction of trapdoors
 - Fast implementation

□ pqNTRUSign: Sign

- Input a hash function, m , (pf, g) , h
 - $(s_p, t_p) = \text{hash}(m|h)$ # hash the message and PK into a mod p vector
 - $r \leftarrow \text{Sampler}$ # generate a mask vector from certain sampler
 - $s_0 = pr + s_p$ # $s_0 \equiv s_p \pmod{p}$
 - $t_0 = s_0 h$ # (s_0, t_0) is a lattice vector
 - $a = (t_p - t_0)g^{-1} \pmod{p}$
 - $(s_1, t_1) = a(pf, g)$ # $t_0 + t_1 \equiv t_p \pmod{p}$
 - $(s, t) = (s_0, t_0) + (s_1, t_1)$
- Repeat above steps with rejection sampling
- Output (s, t)

□ pqNTRUSign: verify

- Input $(s, t), m, h$
 - Check $(s, t) = \text{hash}(m|h) \bmod p$
 - Check $\|(s, t)\|_{\infty}$ is within some bound
 - Check $t = sh$

□ pqNTRUSign: rejection sampling

- Uniform distribution:

- Reject (s, t) when $\|(s, t)\|_\infty > \frac{q}{2} - B$ for some bound B
- High rejection rate, large sig. size $\approx n \log_2 q$

- (bimodal) Gaussian distribution:

- Reject s with probability $e^{\left(\frac{-2\langle(r+af),af\rangle+\|af\|_2}{2\sigma^2}\right)}$
- Reject t when $\|t\|_\infty > \frac{q}{2} - B$ for some bound B
- Low rejection rate, small sig. size $\approx n \left(\frac{\log_2 q}{2} + 2\right)$ with compression

□ pqNTRUSign: signature aggregation

- Store t as the signature, compute $s = t h^{-1}$ during verification
- s follows Gaussian \rightarrow Sum of s will not cause wraparound \rightarrow sum of $s = \text{sum of } s_p \text{ mod } p$
- Verify $k = (q/2\sigma)^2$ signatures in a single ring multiplication
- In practice verify $k \sim 2000$ signatures in 0.3 ms
- At a cost of increasing signature size
 - Gaussian s (4608 bits) vs uniform t (8196 bits)

■ NTRUEncrypt

- Constant time implementation for NTRU-743
 - 128 bits quantum security
 - AVX2
 - Combination of Karatsuba, Toom-3 and Toom-4
 - 2.23x faster than reference implementation



■ Thank you!

■ <https://github.com/NTRUOpenSourceProject>