Revisiting TESLA in the quantum random oracle model

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Selected history of Fiat-Shamir—style signatures from LWE or SIS

2012
Lyubashevsky
Sigs via Fiat-Shamir

2013
Bai-Galbraith
Short sigs

2014
DBGGOPSS
Improvements, fast implementation

2015
TESLA
Tight security reduction, fast implementation

2016
ring-TESLA
Now with rings, fast implementation

TESLA#
Improvements, fast implementation
Selected history of Fiat-Shamir—style signatures from LWE or SIS

- 2012
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    - Improvements, fast implementation

- 2015
  - TESLA
    - Tight security reduction, fast implementation
  - ring-TESLA
    - Now with rings, fast implementation
  - This talk

- 2016
  - TESLA#
    - Improvements, fast implementation
Preamble

- Parameter selection, tightness.
- The quantum random oracle model (QROM).
Given a forger...
...construct a P-solver

\[ \mathcal{P} \text{-solver:} \]

\[ \mathcal{P} \text{ input} \rightarrow \text{Forger} \rightarrow \mathcal{P} \text{ output} \]
Parameter choice should account for the security reduction

\[(\mathcal{P}\text{-}solver \ run \ time)\]
\[= (\text{forger run time}) + (\text{reduction run time})\]

**Assumption:** Problem $\mathcal{P}$ cannot be solved in time less than $t$.

\[(\text{forger run time}) \geq t - (\text{reduction run time})\]

**Important:** Choose parameters so that $t - (\text{reduction run time})$ is intractable.
Tightness

- If (reduction run time) is small then the reduction is tight.

- All else equal, tight is preferred to non-tight:
  - Superior efficiency for a given level of security.
The quantum random oracle model (QROM)

- A quantum forger can query the random oracle in quantum superposition.

- It is conceivable that a scheme is secure in ROM but not in QROM.

- For a scheme to be quantum-resistant, its security reduction must hold in the QROM.
When does ROM imply QROM?

- [BDFLSZ-2011]: ROM $\implies$ QROM if the reduction is history-free.

- Many ROM proofs involve re-programming the random oracle.
  
  - Not history-free.

- There is little research on QROM + re-programming. [Unruh-2014, ES-2015, Unruh-2017]
Prior work on TESLA

Lyubashevsky
Sigs via Fiat-Shamir

Bai-Galbraith
Short sigs

DBGGOPSS
Improvements, fast implementation

TESLA
Tight security reduction, fast implementation

ring-TESLA
Now with rings, fast implementation

TESLA#
Improvements, fast implementation

BLISS
Optimized

Reduction from LWE, SIS.
Proof uses Forking Lemma.
Non-tight, re-programming.
ROM but not QROM.

Reduction from LWE only.
Tight reduction in ROM.
QROM via chameleon hash functions.
Our contributions (theoretical)

• The 2015 TESLA security proof is flawed. (Also noticed by Chris Peikert.)

• New, tight security reduction from LWE.
  – Completely re-done from scratch.

• Direct reduction in the QROM with re-programming.
  – No need for chameleon hash functions.

• Bonus: Proofs of Gaussian heuristic for $q$-ary lattices.
Our contributions (practical)

- New parameter sets chosen according to our tight security reduction.

<table>
<thead>
<tr>
<th>TESLA-0</th>
<th>96 bits classical</th>
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- Software implementation of TESLA-0, TESLA-1 targeting Intel Haswell CPU.
Summary of related work

- Proof approach: [KW-2003], [AFLT-2012].
- Other tight LWE/SIS sigs: [GPV-2008], [BL-2016] (trapdoor), [AFLT-2012].
“Lattice-based” crypto

- **Fact:** LWE/SIS $\geq$ GapSVP, SIVP.

- These are *worst-case to average-case reductions* from fundamental hard problems on lattices.

- However, these reductions are non-tight.

- Parameter sets are never selected according to these reductions.

- Our TESLA parameter sets are based on hardness of LWE, not GapSVP or SIVP.
“Lattice-based” crypto

**TESLA:**
Tightly-secure, Efficient signature scheme from Standard LAttices.

**TESLA:**
Tightly-secure, Efficient Signature scheme from Learning with Arrors.
Learning with Errors (LWE) (matrix version)

**Input.** Matrices $A \in \mathbb{Z}_q^{m \times n}$ and $T \in \mathbb{Z}_q^{m \times n'}$.

- Entries of $A$ are uniformly random.

**Yes.** $T = AS + E$

- Entries of $S \in \mathbb{Z}_q^{n \times n'}$ and $E \in \mathbb{Z}_q^{m \times n'}$ sampled from a discrete Gaussian on $\mathbb{Z}_q$.
- $(S, E)$ is a witness.

**No.** Entries of $T$ are uniformly random.
TESLA key generation

Pk: LWE yes-instance  Sk: witness

1. Choose LWE witness \((S, E)\) with Gaussian entries.
2. Check: If entries of \(S, E\) are too large then restart.
3. Choose \(A\) uniformly at random.
4. \(T \leftarrow AS + E\)
5. Return pk: \((A, T)\), sk: \((S, E)\).
**TESLA sign**

Zero-knowledge proof \((S, E) + \text{Fiat-Shamir}\)

**Input.** sk: \((S, E)\), pk: \((A, T)\), msg.

1. Choose a random “short” vector \(y \in \mathbb{Z}_q^n\).
2. \(c \leftarrow H(\text{hi-bits}(Ay), \text{msg})\).
3. If \(Ay - Ec\) is not “well-rounded” then restart.
4. \(z \leftarrow y + Sc\).
5. If \(z\) is not “short” then restart.
6. Return signature \((z, c)\).
TESLA sign: terminology

- Range of $H(\cdot)$ is vectors in $\{-1, 0, 1\}^{n'}$ of bounded weight.
  - Entries of $Ec, Sc$ are guaranteed to be bounded.

- $w$ is well-rounded if entries of $w$, lo-bits($w$) are not too large.
  - Signatures always verify.
  - Signatures do not leak info on the secret key.
TESLA verify

**Input.** \( (A, T), \) sig: \( (z, c), \) msg.

1. If \( z \) is not “short” then reject.

2. Accept \( \iff c = H(\text{hi-bits}(Az - Tc), \text{msg}) \).

**Observe.**

\[
Az - Tc = Ay - Ec \\
\text{hi-bits}(Ay - Ec) = \text{hi-bits}(Ay)
\]

due to well-roundedness.
Security theorem for TESLA

**Theorem.** Matrix-LWE is $(t, \varepsilon)$-hard $\implies$ TESLA is $(t', \varepsilon')$-unforgeable in QROM with $t' \lesssim t$ and

$$\varepsilon' \leq \varepsilon + \text{(negl in TESLA params)}$$
Security theorem for TESLA

**Theorem.** Matrix-LWE is \((t, \varepsilon)-hard \implies TESLA\) is \((t', \varepsilon')\)-unforgeable in QROM with \(t' \lesssim t\) and

\[
\varepsilon' \leq \varepsilon + (\text{negl in TESLA params})
\]
Proof overview

Follow the lead of [KW-2003], [AFLT-2012]; make it work in QROM.

Suppose there is a TESLA forger:
Simulator

Build a simulator for TESLA signatures.

Real sign, hash oracles.

Simulated oracles.
Forger forges, even with a simulator

If simulation is accurate then

\[
\text{output of (forger + real)} \\
\approx \text{output of (forger + sim)}
\]
Forger + Simulator = LWE solver

LWE solver:

- Simulator
- Forger

LWE instance \((A, T)\)

output “yes” \(\iff\) valid forgery
Forger + Simulator = LWE solver

• If \((A, T)\) is a LWE yes-instance:

\[
\text{forger + simulator} = \text{forgery}
\]

\[\implies \text{output “yes”}.
\]

• \textbf{Need to prove:} If \((A, T)\) is a LWE no-instance:

\[
\text{forger + simulator} \neq \text{forgery}
\]

\[\implies \text{output “no”}.
\]
Yes-instances: Signature simulator

**Input.** \( (A, T) \), msg.

1. \( z \leftarrow \text{random "short" element of } \mathbb{Z}_q^n. \)
2. \( c \leftarrow \text{random } c \in \{ -1, 0, 1 \}^{n'} \text{ of bounded weight.} \)
3. If \( Az - Tc \) is not “well-rounded” then restart.
4. Re-program \( H(\text{hi-bits}(Az - Tc), \text{msg}) \leftarrow c. \)
5. Return signature \( (z, c) \).
Yes-instances: Signature simulator

**Input.** \( \text{pk: } (A, T), \text{ msg.} \)

1. \( z \leftarrow \text{random “short” element of } \mathbb{Z}_q^n. \)

2. \( c \leftarrow \text{random } c \in \{-1, 0, 1\}^{n'} \text{ of bounded weight.} \)

3. If \( Az - Tc \) is not “well-rounded” then restart.

4. **Re-program** \( H(\text{hi-bits}(Az - Tc), \text{msg}) \leftarrow c. \)

5. Return signature \( (z, c). \)
Re-programming in TESLA

$\rho_H$: State prepared with $t$ queries to $H(\cdot)$.

$H'(\cdot)$: Agrees with $H(\cdot)$ except on a small number of inputs $(\cdot, \text{msg})$ for each msg.

**Theorem.** $\|\rho_{H'} - \rho_H\|_1 < \gamma$ except w/Pr

$$\frac{t^2}{\gamma^2} \times (\text{negl in TESLA params}).$$

**Proof.** [BBBV-1996] + Markov’s inequality + gymnastics. $\square$
No-instances: Good hash inputs

• Ability to forge $\mapsto$ can find $(w, \text{msg})$ whose hash 
  \[ c = \text{H}(w, \text{msg}) \] satisfies:

  \[ \exists \text{ short } z \text{ with } \text{hi-bits}(Az - Tc) = w. \quad (1) \]

• $\forall (w, \text{msg})$: The hash of $(w, \text{msg})$ obeys $(1)$ with prob

  \[ \frac{\#\{c \text{ with } (1)\}}{\#\{\text{all } c\}} \]

  over the choice of random oracle $\text{H}(\cdot)$.
Search through unstructured space

- Need to prove: $\#\{c \text{ with } (1)\}$ is small for LWE no-instances.
- Then: each $(w, \text{msg})$ leads to a forgery with negligible probability, independent of all others.
- The only way to find such a $(w, \text{msg})$ is by search through unstructured space.
- Apply lower bounds for quantum search. (Grover is optimal.)
- Forging for LWE no-instances requires many hash queries.
Good hash inputs are rare

**Theorem.** If TESLA params are “convenient” then

$$\operatorname{Ex}_{(A,T)} \left[ \max_w \# \{c \text{ with } (1) \} \right] \leq 1.$$  

**Proof.** Take differences, swap summations. $\square$
Parameter sets

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- Warning: parameters are large, TESLA is not yet ready for practical use.

- Our priority is to establish a correct security reduction in QROM.

- That said, TESLA is far more efficient than all other schemes whose parameter choice accounts for a reduction from LWE/SIS.
## Parameter sets

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<tbody>
<tr>
<td>$n$</td>
<td>644</td>
<td>804</td>
<td>1300</td>
</tr>
<tr>
<td>$n'$</td>
<td>390</td>
<td>600</td>
<td>1036</td>
</tr>
<tr>
<td>$m$</td>
<td>3156</td>
<td>4972</td>
<td>4788</td>
</tr>
<tr>
<td>$q$</td>
<td>$2^{31} - 99$</td>
<td>$2^{31} - 19$</td>
<td>$\approx 2^{36}$</td>
</tr>
<tr>
<td>pk</td>
<td>4.6 MB</td>
<td>11.2 MB</td>
<td>21.8 MB</td>
</tr>
<tr>
<td>sk</td>
<td>1.8 MB</td>
<td>4.2 MB</td>
<td>7.7 MB</td>
</tr>
<tr>
<td>sig</td>
<td>1.8 KB</td>
<td>2.3 KB</td>
<td>4.0 KB</td>
</tr>
</tbody>
</table>

(Key sizes would be much smaller in ring-TESLA.)

\[
A \in \mathbb{Z}_q^{m \times n} \\
E, T \in \mathbb{Z}_q^{m \times n'} \\
S \in \mathbb{Z}_q^{n \times n'} \\
z \in \mathbb{Z}_q^n
\]
Software

- Targets the Intel Haswell microarchitecture.
- Based on software from [DBGGOPSS-2014].
- Use vectorized instructions where possible.
- TESLA-2 params are too big for intermediate computations to fit into a 53-bit mantissa
  - Cannot use the same code as TESLA-{0,1}.

https://tesla.informatik.tu-darmstadt.de/de/tesla/
Thank you!
Global A matrix?

- **Alternative:** Make $A$ a fixed, global parameter.

- **Pros:** Smaller public keys, no expensive seed expansion.

- **Cons:** Potential back door, all-for-the-price-of-one attacks.
Proof approach

- [KW-2003]: Tightly secure signatures from DDH.
- [AFLT-2012]: Transform “lossy” ID schemes into tightly secure signatures.
  - ROM proof involves re-programming. Not history-free. Not known to hold in QROM.
  - TESLA could fit into this framework.
  - Need to re-prove AFLT in the QROM.

Abdalla, Fouque, Lyubashevsky, Tibouchi
Other tightly-secure LWE or SIS signatures (move to the end?)

- [GPV-2008]: Lattice trapdoor. History-free reduction in ROM $\implies$ QROM.
- [BL-2016]: Lattice trapdoor. Standard model (no ROM).
- Trapdoor sigs tend to be inefficient in practice.
- [AFLT-2012]: A variant of the Lyubashevsky scheme.
  - ROM but not QROM (due to AFLT).
  - Still somewhat inefficient.
Comparison: LWE/SIS schemes

- The only other scheme with parameters selected according to a reduction from LWE/SIS is the trapdoor scheme [GPV-2008]. (Parameters and implementation in [BB-2013].)

- Compared to [GPV-2008] at 96-bit classical, 59-bit quantum security:
  - TESLA-2 key sizes 25% smaller.
  - TESLA-2 sig sizes 87% smaller.
  - TESLA-1 cycle counts < 50% smaller.

- Ring-[GVP-2008] is more efficient, but so too would be Ring-TESLA.
Comparison: hash-based schemes

- The only other QR schemes with security reductions in the QROM are hash-based schemes. (e.g. SPHINCS, Leighton-Micali.)
  - TESLA key sizes are much larger.
  - TESLA cycle counts are larger, but could become smaller with future work. (Ring-TESLA.)
  - TESLA signature size is much smaller.