Fault Attacks on Supersingular Isogeny Cryptosystems

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Outline

Definition (Discrete Logarithm Problem)

Pick an abelian group $G = \langle g \rangle$. Given g and X, where $X = g^s$, recover s.

- Each scalar s determines the map $g \mapsto g^s$.
- Fixing s is same as fixing endomorphism $\phi_s : G \to G$.

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Let's generalise this!

- Fix a finite field $k = \mathbb{F}_p$ and a finite extension $K = \mathbb{F}_q$ where $q = p^k$.
- Let E_1 and E_2 be elliptic curves over K.

Definition

An <u>isogeny</u> between E_1 and E_2 is a non-constant morphism defined over $\mathbb{F}_q$ that sends $\mathcal{O}_1$ to $\mathcal{O}_2$. We say that E_1 and E_2 are <u>isogenous</u>.

Isogenies

Fun facts:

- Isogenies are group homomorphisms.
- For every finite subgroup $G \subset E_1$, there is a unique E_2 (up to isomorphism) and a separable $\phi : E_1 \to E_2$ such that $\ker \phi = G$. We write $E_2 = E_1/G$.
- The isogeny can be constructed by an algorithm by Vélu.
- For any $\phi : E \to E'$ of degree n, there exists a unique $\hat{\phi} : E' \to E$ such that $\phi \circ \hat{\phi} = [n] = \hat{\phi} \circ \phi$.
- For any $\phi : E \to E'$ of degree nm, we can decompose ϕ into isogenies of degrees m and n.

Definition

An elliptic curve $E/\mathbb{F}_{p^k}$ is said to be supersingular if $\#E(\mathbb{F}_{p^k}) \equiv 1 \pmod{p}$.

Fun facts:

- All supersingular elliptic curves can be defined over $\mathbb{F}_{p^2}$.
- There are approximately $p/12$ supersingular curves up to isomorphism.

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Definition (Supersingular isogeny problem)

Given two supersingular elliptic curves E_1 and E_2, find an isogeny between them.

Key exchange

Set up:

- Choose $p = 2^n \cdot 3^m \cdot f \pm 1$, such that $2^n \approx 3^m$ and f small.
- Choose supersingular elliptic curve E over $\mathbb{F}_{p^2}$.
- Alice works over $E[2^n]$ with linearly independent points P_A, Q_A.
- Bob works over $E[3^m]$ with linearly independent points P_B, Q_B.

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Recall that

$$E[N] = \mathbb{Z}/N\mathbb{Z} \times \mathbb{Z}/N\mathbb{Z}$$

if N is co-prime to the characteristic of the field.

Key exchange

$$\begin{tikzcd} E \arrow[r, "\phi_A"] \arrow[d, "\phi_B"'] & E/G_A \\ E/G_B & \end{tikzcd}$$

- Picks secret $1 \leq a_1, a_2 \leq 2^n$, not both divisible by 2, which determines $G_A = \langle [a_1]P_A + [a_2]Q_A \rangle$.
- Computes ϕ_A with $\ker \phi_A = G_A$ via Vélu.
- Sends E/G_A, $\phi_A(P_B)$, $\phi_A(Q_B)$.

Key exchange

$$\begin{tikzcd} E \arrow[r, "\phi_A"] \arrow[d, "\phi_B"'] & E/G_A \arrow[d] \\ E/G_B \arrow[r] & E/\langle G_A, G_B \rangle \end{tikzcd}$$

- Receives E/G_B, $\phi_B(P_A)$, $\phi_B(Q_A)$.
- Computes

$$\begin{aligned} G'_A &= \langle [a_1]\phi_B(P_A) + [a_2]\phi_B(Q_A) \rangle \\ &= \langle \phi_B([a_1]P_A + [a_2]Q_A) \rangle \\ &= \phi_B(G_A)\,. \end{aligned}$$

- Uses $j(E_{AB})$ as secret key.

Fault attacks

One can try to find mathematical algorithms to break the cryptosystem. Or, one can use side-channel attacks.

Fault attacks are physical attacks aimed at physical devices and may be induced by:

- EM probe
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Fault attacks cause computation of unintended values which may leak sensitive data.

Fault attacks in ECC

Given elliptic curve E, base point P, compute $[\lambda]P$.

- Introduce fault to base point $P \in E$ to become $P' \in E'$.
  - Change in curves occurs because operation does not use a_6.
- This changes the elliptic curve from E to E' and potentially makes solving ECDLP easier.
- Solving the ECDLP on $[\lambda]P'$ on E', we learn information about λ.

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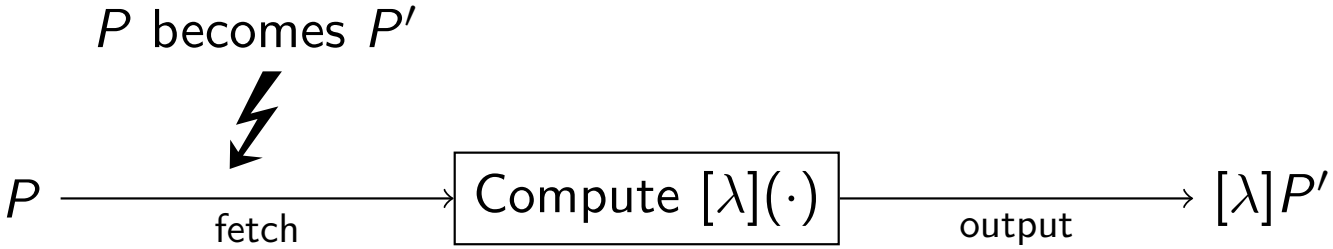

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Fault attacks in Isogenies

Given a point P and an isogeny ϕ, compute $\phi(P)$.

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Given a point P and an isogeny ϕ, compute $\phi(P)$.

- Introduce fault to base point $P \in E$ to become $P' \in E$.
- Compute $[3^m][f]\phi(P')$ to get Z which will have order 2^n with high probability.
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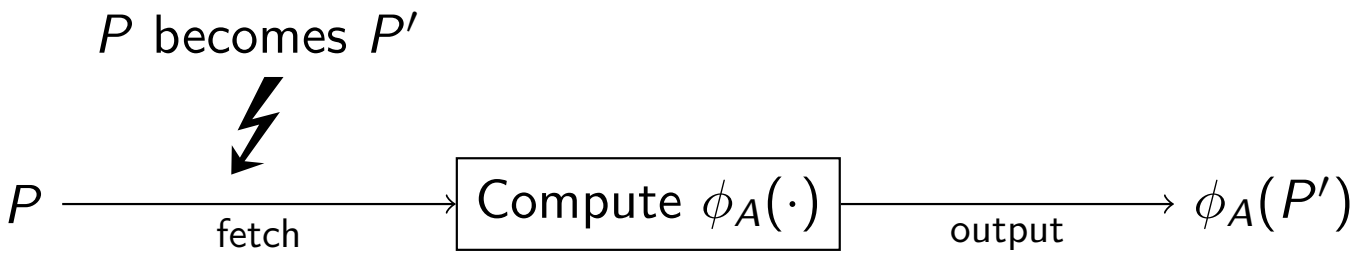

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Faulted point still on curve

- Introduce a fault to the x-coordinate of P.
- Recover P' by solving for y-coordinate. Then P' will lie in E or its quadratic twist E'.
- Some implementations do not distinguish between the two.
- If not, there is a 50% chance of P' landing in E.

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Lemma

Let E_1 be a supersingular elliptic curve over $\mathbb{F}_{p^2}$, where $p = 2^n 3^m f \pm 1$. Suppose $\phi : E_1 \to E_2$ is a separable isogeny of degree 2^n. If $\phi(P') \in E_2$ has order 2^n, then the kernel of $\hat{\phi}$ will be generated by $\phi(P')$.

N.B. $\phi(P')$ does not have to have order 2^n. If order is close to 2^n, we can brute force.

$$\begin{tikzcd} E \arrow[r, "\phi_A"] \arrow[d, "\phi_B"'] & E_A \arrow[d] \\ E_B \arrow[r] & E_{AB} \end{tikzcd}$$

Aim: Recover secret ϕ_A.

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Aim: Recover secret ϕ_A.

- Need to evaluate image of random point under ϕ_A.
- Fault injection before computation of $\phi_A(P_B)$ or $\phi_A(Q_B)$.
- Alice outputs $\phi_A(P')$, hence attacker may recover ϕ_A.

Conclusion

- Image of random points on secret isogeny gives away secret.
  - Recover point of order equal to degree of isogeny.
  - Use point as kernel to construct dual isogeny.
- Important to use countermeasures and checks in implementations!
  - Check point order
  - Able to use point compression in signatures

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THANK YOU!

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Also, thanks to NZMS!